

Confinement Physics in Quantum Chromodynamics

H. Suganuma, H. Ichie, K. Amemiya and A. Tanaka
*Research Center for Nuclear Physics (RCNP), Osaka University
Ibaraki, Osaka 567-0047, Japan
E-mail: suganuma@rcnp.osaka-u.ac.jp*

We study the confinement physics in QCD in the maximally abelian (MA) gauge using the SU(2) lattice QCD, based on the dual-superconductor picture. In the MA gauge, off-diagonal gluon components are forced to be small, and the off-diagonal angle variable $\chi_\mu(s)$ tends to be random. Within the random-variable approximation for $\chi_\mu(s)$, we analytically prove the perimeter law of the off-diagonal gluon contribution to the Wilson loop in the MA gauge, which leads to abelian dominance on the string tension. To clarify the origin of abelian dominance for the long-range physics, we study the charged-gluon propagator in the MA gauge using the lattice QCD, and find that the effective mass $m_{ch} \simeq 0.9\text{GeV}$ of the charged gluon is induced by the MA gauge fixing. In the MA gauge, there appears the macroscopic network of the monopole world-line covering the whole system, which would be identified as monopole condensation at a large scale. To prove monopole condensation in the field-theoretical manner, we derive the inter-monopole potential from the dual Wilson loop in the monopole part of QCD, which carries the nonperturbative QCD aspects, in the MA gauge. The dual gluon mass is evaluated as $m_B \simeq 0.5\text{GeV}$ in the monopole part in the infrared region, which is the evidence of the dual Higgs mechanism by monopole condensation.

1 QCD and Dual Superconductor Picture for Confinement

Quantum chromodynamics (QCD) is the fundamental theory of the strong interaction. In spite of the simple form of the QCD lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{tr}G_{\mu\nu}G^{\mu\nu} + \bar{q}(i\cancel{D} - m_q)q, \quad (1)$$

it miraculously provides quite various phenomena like color confinement, chiral symmetry breaking, nontrivial topologies, quantum anomalies and so on. It would be interesting to compare QCD with the history of the Universe, because a quite simple ‘Big Bang’ also created everything including galaxies, stars and living things. Therefore, QCD can be regarded as an interesting miniature of the Universe. This is the most attractive point of the QCD physics. In this paper, we will slightly touch the confinement physics in QCD.

The Regge trajectory of hadrons and the lattice QCD show that the confinement force between the color-electric charges is characterized by the universal physical quantity of the string tension $\sigma \simeq 1\text{GeV/fm}$ and is brought by the *one-dimensional squeezing* of the color-electric flux in the QCD vacuum.

Based on the electro-magnetic duality, Nambu proposed the *dual superconductor picture* for quark confinement in 1974.¹ In this picture, such a squeezing of the color-electric flux between quarks is realized by the *dual Meissner effect*, as the result of *color-magnetic monopole condensation*, which is the dual version of electric-charge condensation in the superconductor. However, there are *two large gaps* between QCD and the dual-superconductor picture.²

- (1) This picture is based on the abelian gauge theory subject to the Maxwell-type equations, where electro-magnetic duality is manifest, while QCD is a nonabelian gauge theory.
- (2) The dual-superconductor scenario requires condensation of (color-) magnetic monopoles as the key concept, while QCD does not have such a monopole as the elementary degrees of freedom.

These gaps can be simultaneously fulfilled by the use of the '*t Hooft abelian gauge fixing*', the partial gauge fixing which only remains abelian gauge degrees of freedom in QCD.³ The abelian gauge fixing reduces QCD to an abelian gauge theory, where the off-diagonal gluon behaves as a charged matter field similar to W_μ^\pm in the Standard Model and provides a color-electric current in terms of the residual abelian gauge symmetry. As a remarkable fact in the abelian gauge, *color-magnetic monopoles* appear as *topological objects* corresponding to the nontrivial homotopy group $\Pi_2(\mathrm{SU}(N_c)/\mathrm{U}(1)^{N_c-1}) = \mathbf{Z}_\infty^{N_c-1}$ in a similar manner to the GUT monopole.^{3–5}

Here, let us consider the appearance of monopoles in terms of the gauge connection.^{2,6} In the general system including the singularity such as the Dirac string, the field strength is defined as

$$G_{\mu\nu} \equiv \frac{1}{ie}([\hat{D}_\mu, \hat{D}_\nu] - [\hat{\partial}_\mu, \hat{\partial}_\nu]), \quad (2)$$

which takes a form of the difference between the covariant derivative operator $\hat{D}_\mu \equiv \hat{\partial}_\mu + ieA_\mu(x)$ and the derivative operator $\hat{\partial}_\mu$ satisfying $[\hat{\partial}_\mu, f(x)] = \partial_\mu f(x)$. By the general gauge transformation with the gauge function Ω , \hat{D}_μ is transformed as $\hat{D}_\mu \rightarrow \hat{D}'_\mu \equiv \Omega \hat{D}_\mu \Omega^\dagger$, and $G_{\mu\nu}$ is transformed as

$$\begin{aligned} G_{\mu\nu} \rightarrow G'_{\mu\nu} &\equiv \Omega G_{\mu\nu} \Omega^\dagger = \frac{1}{ie}([\hat{D}'_\mu, \hat{D}'_\nu] - \Omega[\hat{\partial}_\mu, \hat{\partial}_\nu]\Omega^\dagger) \\ &= \partial_\mu A'_\nu - \partial_\nu A'_\mu + ie[A'_\mu, A'_\nu] + \frac{i}{e}\Omega[\partial_\mu, \partial_\nu]\Omega^\dagger. \end{aligned} \quad (3)$$

The last term remains only for the *singular gauge transformation*, and can provide the Dirac string in the abelian gauge sector. For a singular $\mathrm{SU}(N_c)$ gauge function, the last term leads to breaking of the abelian Bianchi identity and monopoles in the abelian gauge.^{2,6}

Thus, QCD in the abelian gauge is an abelian gauge theory including both the electric current j_μ and the magnetic current k_μ , and can provide the theoretical basis of the dual-superconductor scheme for the confinement mechanism.

2 Maximally Abelian Gauge, Abelian Dominance and Global Network of Monopole Current in MA Gauge in Lattice QCD

In the Euclidean QCD, the maximally abelian (MA) gauge is defined by minimizing

$$R_{\text{off}}[A_\mu(\cdot)] \equiv \int d^4x \text{tr}[\hat{D}_\mu, \vec{H}][\hat{D}_\mu, \vec{H}]^\dagger = \frac{e^2}{2} \int d^4x \sum_\alpha |A_\mu^\alpha(x)|^2, \quad (4)$$

where $\hat{D}_\mu \equiv \hat{\partial}_\mu + ieA_\mu$ denotes the $SU(N_c)$ covariant derivative and the Cartan decomposition $A_\mu(x) = \vec{A}_\mu(x) \cdot \vec{H} + \sum_\alpha A_\mu^\alpha(x)E^\alpha$ is used. In the MA gauge, the off-diagonal gluon component is forced to be as small as possible by the gauge transformation, and therefore the gluon field $A_\mu(x) \equiv A_\mu^a(x)T^a$ closely resembles the abelian gauge field $\vec{A}_\mu(x) \cdot \vec{H}$.^{2,6}

In the MA gauge, $G \equiv SU(N_c)_{\text{local}}$ is reduced into $H \equiv U(1)_{\text{local}}^{N_c-1} \times \text{Weyl}_{\text{global}}^{N_c}$, where the global Weyl symmetry is the subgroup of $SU(N_c)$ relating the permutation of the N_c bases.⁷ Since the covariant derivative \hat{D}_μ obeys the adjoint gauge transformation, the MA gauge fixing condition is found to be²

$$[\vec{H}, [\hat{D}_\mu, [\hat{D}_\mu, \vec{H}]]] = 0, \quad (5)$$

which leads the partial gauge fixing on G/H .

In the Euclidean lattice formalism, the MA gauge is defined by maximizing the diagonal element of the link variable $U_\mu(s) \equiv \exp\{iaeA_\mu(s)\}$,^{2,8}

$$R_{\text{diag}}[U_\mu(\cdot)] \equiv \sum_{s,\mu} \text{tr}\{U_\mu(s)\vec{H}U_\mu^\dagger(s)\vec{H}\}. \quad (6)$$

The $SU(N_c)$ link variable is factorized corresponding to the Cartan decomposition $G/H \times H$ as $U_\mu(s) = M_\mu(s)u_\mu(s)$ with $M_\mu(s) \equiv \exp\{i\Sigma_\alpha\theta_\mu^\alpha(s)E^\alpha\}$ and $u_\mu(s) \equiv \exp\{i\vec{\theta}_\mu(s) \cdot \vec{H}\}$. Here, the *abelian link variable* $u_\mu(s) \in H \equiv U(1)^{N_c-1}$ behaves as the abelian gauge field, and the off-diagonal factor $M_\mu(s) \in G/H$ behaves as the charged matter field in terms of the residual abelian gauge symmetry $U(1)_{\text{local}}^{N_c-1}$. In the lattice formalism, the abelian projection is defined by the replacement as $U_\mu(s) \in G \rightarrow u_\mu(s) \in H$.

Abelian dominance and monopole dominance for NP-QCD (confinement⁹, D χ SB¹⁰, instantons^{7,11}) are the remarkable facts observed in the lattice QCD

in the MA gauge. Here, we summarize the QCD system in the MA gauge in terms of abelian dominance, monopole dominance and extraction of the relevant mode for NP-QCD.

- (a) Without gauge fixing, it is difficult to extract relevant degrees of freedom for NP-QCD. All the gluon components equally contribute to NP-QCD.
- (b) In the MA gauge, QCD is reduced into an abelian gauge theory including the electric current j_μ and the magnetic current k_μ . The diagonal gluon $\vec{A}_\mu \cdot \vec{H}$ behaves as the abelian gauge field, and the off-diagonal gluon behaves as the charged matter field in terms of the residual abelian gauge symmetry. In the MA gauge, the lattice QCD shows *abelian dominance* for NP-QCD: only the diagonal gluon is relevant for NP-QCD, while off-diagonal gluons do not contribute to NP-QCD. In the confinement phase of the lattice QCD, there appears the *global network of the monopole world-line covering the whole system in the MA gauge* as shown in Fig.1.
- (c) The diagonal gluon can be decomposed into the “photon part” and the “monopole part”, corresponding to the separation of j_μ and k_μ . The monopole part carries the monopole current k_μ only, *i.e.* $j_\mu \simeq 0$. The photon part carries the electric current j_μ only, *i.e.* $k_\mu \simeq 0$. In the MA gauge, the lattice QCD shows *monopole dominance* for NP-QCD: the monopole part leads to NP-QCD, while the photon part seems trivial like QED and does not contribute to NP-QCD.

Thus, monopoles in the MA gauge can be regarded as the relevant collective mode for NP-QCD, and *formation of the global network of the monopole current seems to mean “monopole condensation” in the infrared scale*.^{2,12}

(a) (b)

Figure 1: The monopole world-line projected into \mathbf{R}^3 in the MA gauge in the SU(2) lattice QCD with $16^3 \times 4$. (a) confinement phase ($\beta = 2.2$), (b) deconfinement phase ($\beta = 2.4$). There appears the global network of monopole currents in the confinement phase.

3 Analytical Proof of Abelian Dominance for Confinement

As long as confinement, monopole dominance seems trivial if abelian dominance holds, because the electric current j_μ does not contribute to the electric confinement. In the confinement physics, the nontrivial phenomenon observed in the MA gauge is abelian dominance. In this section, we analytically prove abelian dominance on the string tension in the MA gauge, considering the off-diagonal gluon properties.^{2,6} In the lattice formalism, the SU(2) link variable is factorized as $U_\mu(s) = M_\mu(s)u_\mu(s)$, according to the Cartan decomposition

$G \simeq G/H \times H$. Here, $u_\mu(s) \equiv \exp\{i\tau^3\theta_\mu^3(s)\} \in H$ denotes the abelian link variable, and the off-diagonal factor $M_\mu(s) \in G/H$ is parameterized as

$$M_\mu(s) \equiv e^{i\{\tau^1\theta_\mu^1(s)+\tau^2\theta_\mu^2(s)\}} = \begin{pmatrix} \cos\theta_\mu(s) & -\sin\theta_\mu(s)e^{-i\chi_\mu(s)} \\ \sin\theta_\mu(s)e^{i\chi_\mu(s)} & \cos\theta_\mu(s) \end{pmatrix}. \quad (7)$$

In the MA gauge, the diagonal element $\cos\theta_\mu(s)$ in $M_\mu(s)$ is maximized by the gauge transformation as large as possible; for instance, the abelian projection rate is almost unity as $R_{\text{Abel}} = \langle \cos\theta_\mu(s) \rangle_{\text{MA}} \simeq 0.93$ at $\beta = 2.4$. Then, the MA gauge fixing provides the two remarkable properties on the off-diagonal element $e^{i\chi_\mu(s)} \sin\theta_\mu(s)$ in $M_\mu(s)$.

- (1) the off-diagonal amplitude $|\sin\theta_\mu(s)|$ is forced to be small in the MA gauge, which allows the approximate treatment on the off-diagonal element.
- (2) the off-diagonal angle variable $\chi_\mu(s)$ is not constrained by the MA gauge-fixing condition at all, and tends to be a random variable.

Hence, $\chi_\mu(s)$ can be regarded as a *random angle variable* on the treatment of $M_\mu(s)$ in the MA gauge in a good approximation.^{2,6}

In calculating the Wilson loop $\langle W_C[U_\mu(\cdot)] \rangle = \langle \text{tr}\Pi_C\{M_\mu(s)u_\mu(s)\} \rangle$, we take the random-variable approximation for the off-diagonal angle variable $\chi_\mu(s)$ in the MA gauge, and then the integral of $e^{i\chi_\mu(s)}$ on $\chi_\mu(s)$ vanishes as

$$\langle e^{i\chi_\mu(s)} \rangle \simeq \int_0^{2\pi} d\chi_\mu(s) \exp\{i\chi_\mu(s)\} = 0. \quad (8)$$

Thus, the *off-diagonal factor* $M_\mu(s)$ appearing in $\langle W_C[U_\mu(\cdot)] \rangle$ is simply reduced as a *c-number factor*, $M_\mu(s) \rightarrow \cos\theta_\mu(s) \mathbf{1}$, and therefore the SU(2) link variable $U_\mu(s)$ in $\langle W_C[U_\mu(\cdot)] \rangle$ is reduced to a *diagonal matrix*,

$$U_\mu(s) \equiv M_\mu(s)u_\mu(s) \rightarrow \cos\theta_\mu(s)u_\mu(s). \quad (9)$$

Then, for the $I \times J$ rectangular C , the Wilson loop $W_C[U_\mu(\cdot)]$ in the MA gauge is approximated as

$$\begin{aligned} \langle W_C[U_\mu(\cdot)] \rangle &\equiv \langle \text{tr}\Pi_{i=1}^L U_{\mu_i}(s_i) \rangle \simeq \langle \Pi_{i=1}^L \cos\theta_{\mu_i}(s_i) \cdot \text{tr}\Pi_{j=1}^L u_{\mu_j}(s_j) \rangle_{\text{MA}} \\ &\simeq \langle \exp\{\sum_{i=1}^L \ln(\cos\theta_{\mu_i}(s_i))\} \rangle_{\text{MA}} \langle W_C[u_\mu(\cdot)] \rangle_{\text{MA}}, \end{aligned} \quad (10)$$

where $L \equiv 2(I+J)$ denotes the perimeter length and $W_C[u_\mu(\cdot)] \equiv \text{tr}\Pi_{i=1}^L u_{\mu_i}(s_i)$ the abelian Wilson loop. Replacing $\sum_{i=1}^L \ln\{\cos(\theta_{\mu_i}(s_i))\}$ by its average $L\langle \ln\{\cos\theta_\mu(\cdot)\} \rangle_{\text{MA}}$ in a statistical sense, we derive a simple estimation as^{2,6}

$$W_C^{\text{off}} \equiv \langle W_C[U_\mu(\cdot)] \rangle / \langle W_C[u_\mu(\cdot)] \rangle_{\text{MA}} \simeq \exp\{L\langle \ln(\cos\theta_\mu(\cdot)) \rangle_{\text{MA}}\} \quad (11)$$

for the *contribution of the off-diagonal gluon element to the Wilson loop*. From this analysis, the contribution of off-diagonal gluons to the Wilson loop is expected to obey the *perimeter law* in the MA gauge for large loops, where the statistical treatment would be accurate.

We show the off-diagonal contribution $W_C^{\text{off}} \equiv \langle W[U_\mu(\cdot)] \rangle / \langle W[u_\mu(\cdot)] \rangle_{\text{MA}}$ to the Wilson loop in the lattice QCD simulation with $\beta = 2.4$ in Fig.2. In the MA gauge, W_C^{off} seems to obey the *perimeter law* for the large loop, and is well reproduced by the estimation of Eq.(11) with the *microscopic input* as $\langle \ln\{\cos\theta_\mu(s)\} \rangle_{\text{MA}} \simeq -0.082$ for $\beta = 2.4$. From Eq.(11), the off-diagonal contribution to the string tension vanishes as

$$\sigma_{\text{SU}(2)} - \sigma_{\text{Abel}} \simeq -2\langle \ln\{\cos\theta_\mu(s)\} \rangle_{\text{MA}} \lim_{R,T \rightarrow \infty} \frac{R+T}{RT} = 0. \quad (12)$$

Thus, *abelian dominance for the string tension*, $\sigma_{\text{SU}(2)} = \sigma_{\text{Abel}}$, can be proved in the MA gauge within the random-variable approximation for the off-diagonal angle variable $\chi_\mu(s)$, although the *finite size effect* on R and T in the Wilson loop leads to the deviation between $\sigma_{\text{SU}(2)}$ and σ_{Abel} as $\sigma_{\text{SU}(2)} > \sigma_{\text{Abel}}$.^{2,6}

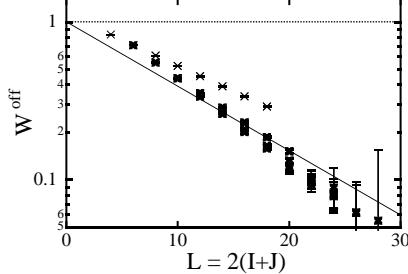


Figure 2: The comparison between the lattice data and the analytical estimation of $W_C^{\text{off}} \equiv \langle W_C[U_\mu(\cdot)] \rangle / \langle W_C[u_\mu(\cdot)] \rangle$ as the function of the perimeter $L \equiv 2(I + J)$ in the MA gauge. The cross (\times) denotes the lattice date at $\beta = 2.4$, and the straight line denotes the theoretical estimation of $W_C^{\text{off}} = \exp\{L\langle \ln\{\cos\theta_\mu(s)\} \rangle_{\text{MA}}\}$ with the microscopic input $\langle \ln\{\cos\theta_\mu(s)\} \rangle_{\text{MA}} \simeq -0.082$ at $\beta = 2.4$. The off-diagonal gluon contribution W_C^{off} seems to obey the *perimeter law* for $I, J \geq 2$.

4 Origin of Abelian Dominance : Effective Charged-Gluon Mass induced in MA Gauge

In this section, we study the origin of abelian dominance for NP-QCD in the MA gauge in terms of the *generation of the effective mass* m_{ch} of the

off-diagonal (charged) gluon by the MA gauge fixing in the QCD generating functional as^{2,12}

$$Z_{\text{QCD}}^{\text{MA}} = \int DA_\mu \exp\{iS_{\text{QCD}}[A_\mu]\} \delta(\Phi_{\text{MA}}^\pm[A_\mu]) \Delta_{\text{FP}}[A_\mu] \\ \simeq \int DA_\mu^3 \exp\{iS_{\text{eff}}[A_\mu^3]\} \int DA_\mu^\pm \exp\{i \int d^4x m_{ch}^2 A_\mu^+ A_\mu^-\} \mathcal{F}[A_\mu] \quad (13)$$

where Δ_{FP} is the Faddeev-Popov determinant, $S_{\text{eff}}[A_\mu^3]$ the abelian effective action and $\mathcal{F}[A_\mu]$ a smooth functional. If the MA gauge fixing induces the effective mass m_{ch} of off-diagonal (charged) gluons, the charged gluon propagation is limited within the short-range region as $r \lesssim m_{ch}^{-1}$, and hence off-diagonal gluons cannot contribute to the long-distance physics in the MA gauge, which provides the origin of abelian dominance for NP-QCD.

Here, using the SU(2) lattice QCD in the Euclidean metric, we study the gluon propagator $G_{\mu\nu}^{ab}(x - y) \equiv \langle A_\mu^a(x) A_\nu^b(y) \rangle$ in the MA gauge.^{2,13} As for the residual U(1)₃ gauge symmetry, we impose the U(1)₃ Landau gauge fixing to extract most continuous gauge configuration and to compare with the continuum theory. The gluon field $A_\mu^a(x)$ is extracted from the link variable as $U_\mu(s) = \exp(iae A_\mu^a(s) \frac{r^a}{2})$. Here, the scalar combination $G_{\mu\mu}^a(r) \equiv \sum_{\mu=1}^4 \langle A_\mu^a(r) A_\mu^a(0) \rangle$ ($a = 1, 2, 3$) is useful to observe the interaction range of the gluon, because it depends only on the four-dimensional Euclidean radial coordinate $r \equiv (x_\mu x_\mu)^{\frac{1}{2}}$.

We calculate the gluon propagator $G_{\mu\mu}^a(r)$ in the MA gauge using the SU(2) lattice QCD with $12^3 \times 24$ and $2.2 \leq \beta \leq 2.4$. In the MA gauge, the off-diagonal (charged) gluon propagates only within the short-range region $r \lesssim 0.4$ fm, so that it cannot contribute to the long-range physics. On the other hand, the diagonal gluon propagates over the long distance and influences the long-range physics. Thus, we find *abelian dominance for the gluon propagator in the MA gauge*, and this is the *origin of abelian dominance for the long-distance physics or NP-QCD*.^{2,13}

Since the propagator of the massive vector boson with mass M asymptotically behaves as the Yukawa-type function $G_{\mu\mu}(r) \simeq \frac{3\sqrt{M}}{(2\pi)^{3/2}} \cdot \frac{\exp(-Mr)}{r^{3/2}}$, the effective mass m_{ch} of the charged gluon can be evaluated from the slope of the logarithmic plot of $r^{3/2} G_{\mu\mu}^{+-}(r) \sim \exp(-m_{ch} r)$ as shown in Fig.3. The charged gluon behaves as a massive particle at the long distance, $r \gtrsim 0.4$ fm. We obtain the *effective mass of the charged gluon* as $m_{ch} \simeq 0.94$ GeV,^{2,13} which provides the *critical scale on abelian dominance*.

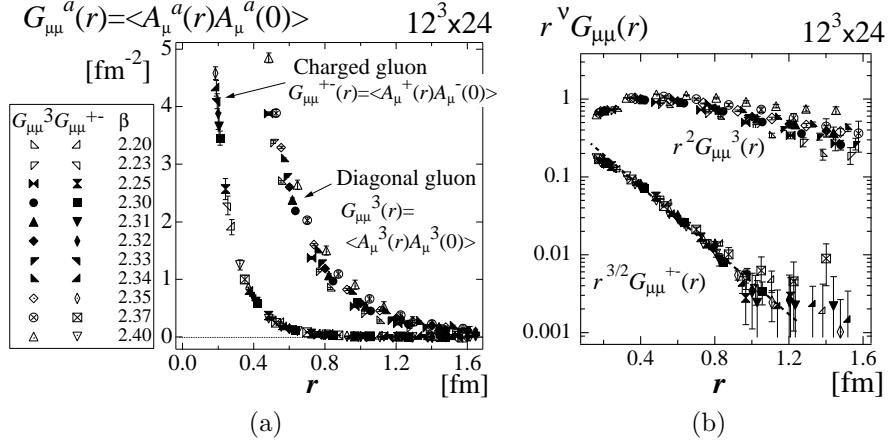


Figure 3: (a) The scalar correlation $G_{\mu\mu}^a(r)$ of the gluon propagator as the function of the 4-dimensional distance r in the MA gauge in the SU(2) lattice QCD with $12^3 \times 24$ and $2.2 \leq \beta \leq 2.4$. In the MA gauge, the off-diagonal (charged) gluon propagates within the short-range region $r \lesssim 0.4$ fm, and cannot contribute to the long-range physics. (b) The logarithmic plot for the scalar correlation $r^\nu G_{\mu\mu}^a(r)$. The charged-gluon propagator behaves as the Yukawa-type function, $G_{\mu\mu} \sim \frac{\exp(-m_{ch}r)}{r^{3/2}}$. The effective mass of the charged gluon can be estimated as $m_{ch} \simeq 0.94$ GeV from the slope of the dotted line.

5 Dual Wilson Loop, Inter-Monopole Potential and Evidence of Dual Higgs Mechanism (Monopole Condensation)

In this section, we study the dual Higgs mechanism by monopole condensation in the NP-QCD vacuum in the field-theoretical manner. Since QCD is described by the “electric variable” as quarks and gluons, the “electric sector” of QCD has been well studied with the Wilson loop or the inter-quark potential, however, the “magnetic sector” of QCD is hidden and still unclear. To investigate the magnetic sector directly, it is useful to introduce the “dual (magnetic) variable” as the *dual gluon* B_μ , similarly in the dual Ginzburg-Landau (DGL) theory.^{5,14–17} The dual gluon B_μ is the dual partner of the diagonal gluon and directly couples with the magnetic current k_μ .

Here, we concentrate ourselves to the monopole part in the MA gauge, which holds the essence of NP-QCD. Since the monopole part does not include the electric current as $\partial_\mu F^{\mu\nu} = j^\nu \simeq 0$, the dual gluon B_μ can be introduced as the regular field satisfying $\partial_\mu B_\nu - \partial_\nu B_\mu = {}^*F_{\mu\nu}$ and the dual Bianchi identity, $\partial^\mu ({}^*(\partial \wedge B))_{\mu\nu} = 0$. In terms of the dual Higgs mechanism, the inter-monopole potential is expected to be short-range Yukawa-type, and the dual gluon B_μ

becomes massive in the monopole-condensed vacuum. To examine the inter-monopole potential, we define *the dual Wilson loop* W_D as the line-integral of the dual gluon B_μ along a loop C ,^{2,12,18}

$$W_D(C) \equiv \exp\left\{i\frac{e}{2} \oint_C dx_\mu B^\mu\right\} = \exp\left\{i\frac{e}{2} \iint d\sigma_{\mu\nu} {}^*F^{\mu\nu}\right\}, \quad (14)$$

which is the *dual version of the abelian Wilson loop* $W_{\text{Abel}}(C) \equiv \exp\left\{i\frac{e}{2} \oint_C dx_\mu A^\mu\right\} = \exp\left\{i\frac{e}{2} \iint d\sigma_{\mu\nu} F^{\mu\nu}\right\}$. Here, we have set the test monopole charge as $e/2$ considering the duality correspondence. The potential between the monopole and the anti-monopole is derived from the dual Wilson loop as

$$V_M(R) = -\lim_{T \rightarrow \infty} \frac{1}{T} \ln \langle W_D(R, T) \rangle. \quad (15)$$

Using the SU(2) lattice QCD in the MA gauge, we study the dual Wilson loop and the inter-monopole potential in the monopole part. The dual Wilson loop $\langle W_D(R, T) \rangle$ seems to obey the *perimeter law* as $\langle W_D(R, T) \rangle \sim \exp\{-c(R+T)\}$ for large R, T . As shown in Fig.4, the inter-monopole potential is short-ranged and flat in comparison with the inter-quark potential.

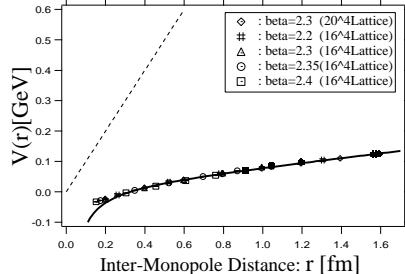


Figure 4: The inter-monopole potential $V_M(r)$ extracted from the dual Wilson loop $\langle W_D(I, J) \rangle$ in the monopole part in the MA gauge. Here, r denotes the 3-dimensional distance between the monopole and the anti-monopole. For comparison, we add the linear inter-quark potential denoted by the dashed line. The solid curve denote the Yukawa potential adding the finite-size effect of the dual Wilson loop.

Except for the short distance, the inter-monopole potential can be fitted by the Yukawa potential $V_M(r) = -\frac{(e/2)^2}{4\pi} \frac{e^{-m_B r}}{r} + \frac{cr}{T_a}$, where the second term is the correction appearing as the finite-size effect of the dual Wilson loop. The dual gluon mass is estimated as $m_B \simeq 0.5$ GeV, which is consistent with the DGL theory.^{5,14–17} *The mass generation of the dual gluon B_μ can be regarded*

as the direct evidence of the dual Higgs mechanism by monopole condensation at the infrared scale in the NP-QCD vacuum.^{2,12,18}

Thus, the lattice QCD in the MA gauge exhibits abelian dominance and monopole condensation in the infrared region, and leads to the dual Ginzburg-Landau theory as the infrared effective theory directly based on QCD^{2,12,19} (See Fig.5).

Acknowledgments

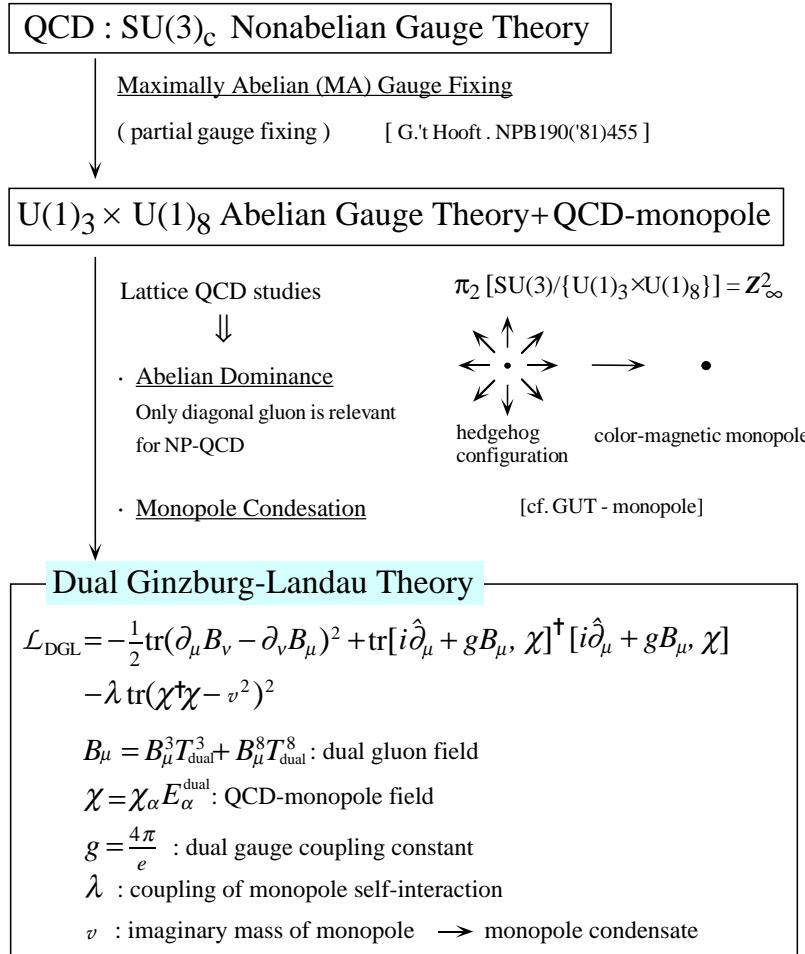
H.S. and H.I. acknowledge Dr. K.Tsushima for his warm hospitality at CSSM.

References

1. Y. Nambu, *Phys. Rev.* **D10**, 4262 (1974).
2. H. Suganuma, H. Ichie, A. Tanaka and K. Amemiya, *Prog. Theor. Phys. Suppl.* **131**, 559 (1998) : hep-lat/9804027.
3. G. 't Hooft, *Nucl. Phys.* **B190**, 455 (1981).
4. Z. F. Ezawa and A. Iwazaki, *Phys. Rev.* **D25**, 2681 ; **D26**, 631 (1982).
5. H. Suganuma, S. Sasaki and H. Toki, *Nucl. Phys.* **B435**, 207 (1995).
6. H. Ichie and H. Suganuma, this Proceedings; Proc. of *Innovative Computational Methods in Nuclear Many-Body Problems (INNOCOM '97)* (World Scientific, 1998) in press : hep-lat/9802032.
7. H. Suganuma, M. Fukushima, H. Ichie and A. Tanaka, *Nucl. Phys.* **B** (Proc. Suppl.) **65**, 29 (1998); H. Suganuma, A. Tanaka, S. Sasaki and O. Miyamura, *Nucl. Phys.* **B** (Proc. Suppl.) **47**, 302 (1996).
8. A. Kronfeld, G. Schierholz and U. Wiese, *Nucl. Phys.* **B293**, 461 (1987).
9. T. Suzuki and I. Yotsuyanagi, *Phys. Rev.* **D42**, 4257 (1990).
10. O. Miyamura, *Phys. Lett.* **B353**, 91 (1995).
11. H. Suganuma, S. Umisedo, S. Sasaki, H. Toki and O. Miyamura, *Aust. J. Phys.* **50**, 233 (1997) : hep-ph/9602368.
12. H. Suganuma, K. Amemiya, A. Tanaka and H. Ichie, in *INNOCOM '97*.
13. K. Amemiya and H. Suganuma, in *INNOCOM '97* : hep-lat/9712028.
14. H. Ichie, H. Suganuma, H. Toki, *Phys. Rev.* **D52**, 2994; **D54**, 3382 ('96).
15. S. Sasaki, H. Suganuma and H. Toki, *Phys. Lett.* **B387**, 145 (1996).
16. S. Umisedo, H. Suganuma and H. Toki, *Phys. Rev.* **D57**, 1605 (1998).
17. H. Monden, H. Ichie, H. Suganuma, H. Toki, *Phys. Rev.* **C57**, 2564 (1998).
18. A. Tanaka and H. Suganuma, in *INNOCOM '97* : hep-lat/9712027.
19. K.-I. Kondo, *Phys. Rev.* **D57**, 7467 (1998).

From Lattice QCD to Dual Ginzburg-Landau Theory

— Infrared Effective Theory directly based on QCD —



Dual Gauge Symmetry is spontaneously broken instead of Gauge Symmetry

Figure 5: Construction of the dual Ginzburg-Landau (DGL) theory from the lattice QCD in the maximally abelian (MA) gauge.